

A Longitudinal Assessment of Antecedent Course Work in Mathematics and Subsequent Mathematical Attainment

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ABSTRACT Using six waves of data (Grades 7–12) from the Longitudinal Study of American Youth, the author examined the effects of different mathematics course work (pre-algebra, geometry, calculus) on subsequent achievement in, and attitude toward, mathematics, with partial adjustment for student background characteristics. Results showed that in the early grades of high school, algebra courses significantly affected mathematics achievement. Mathematics course work, however, did not play a significant role in mathematics achievement in the middle grades of high schools. There was a “harvest” of significant course-work indicators in the later grades of high school; every advanced mathematics course affected mathematics achievement. Many course-work effects were substantial even after accounting for variables such as prior mathematics achievement and socioeconomic status.

Consistent with conventional wisdom, time spent on mathematics and course work in advanced mathematics have emerged as two powerful predictors of mathematics achievement from large-scale, nationally representative studies. Mathematics course work shows a significant effect on mathematics achievement even after a partial adjustment for student demographic characteristics,¹ which indicates that the effect of course work on achievement in mathematics is over and above the effect of student background (Hoffer, Rasinski, & Moore, 1995; Myers & Milne, 1988). Of the five academic areas included in the High School and Beyond (HS&B) survey (vocabulary, reading, writing, mathematics, and science), mathematics is the most sensitive to additional course work and school graduation (Rock, Ekstrom, Goertz, & Pollack, 1986).

The National Assessment of Educational Progress (NAEP) also reported substantial achievement in mathematics associated with advanced mathematics courses (Dossey, Mullis, Lindquist, & Chambers, 1988). At age 17, students who took pre-algebra as their most advanced course scored 272 points on a 500-point scale that measured

cognitive skills (e.g., concepts, problem solving) in various content areas (e.g., algebra, geometry) in comparison with 287 points for Algebra I, 301 points for geometry, 320 points for Algebra II, and 343 points for pre-calculus (Dossey et al., 1988). Other researchers using nationally representative data reported that high school students who take more mathematics courses perform better in standardized tests of mathematics achievement (Gamoran, 1987; Hoffer et al., 1995; Rock & Pollack, 1995; Sebring, 1987).

Witte (1992) concluded that the effectiveness of students' additional course work is the most solid policy implication that can be drawn from large-scale national studies. Similar evidence has cumulated over the past decade, partly leading several states to raise the mathematics requirement (from 2 to 3 years) for high school graduation in the late 1980s.

Unfortunately, many schools coped with the higher requirement by offering more low-level mathematics courses because of, for example, a lack of manpower for advanced courses (Hoffer, 1997). Many students took more low-level mathematics courses to meet the increased graduation requirement (Goertz, 1989; Patterson, 1991). As a result, the higher requirement has diluted the effects of additional mathematics courses on mathematics achievement (Clune & White, 1992; Porter, 1995; Wilson & Rossman, 1993). Hoffer (1997) concluded that “requiring students to complete three years of mathematics appears neither to raise average math achievement levels, nor to reduce the impact of SES [socioeconomic status] background on high school learning outcomes” (p. 596). A similar argument is supported by the fact that American students still fall far behind the international average in mathematics in the Third International Mathematics and Science Studies (TIMSS; Beaton et

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al., 1996), even though student enrollment in mathematics has been improving constantly in the 1990s (National Center for Education Statistics, 1995).

The effect of mathematics course work caught the attention of researchers in the 1970s when studies of gender differences in mathematics achievement shifted from the biological perspective to the sociological perspective. Researchers showed that gender differences in mathematics course work contributed to the achievement gap in mathematics between females and males (Fennema, 1977, 1979, 1980; Fennema & Sherman, 1977a, 1977b; Wise, Steel, & MacDonald, 1979). Most researchers have become aware that different course-taking patterns can be a potential factor in the study of mathematics achievement.

In the current cycle of education reform, researchers have begun to pay increased attention to affective factors in mathematics learning, particularly attitude toward mathematics (McLeod, 1992). There are significant relationships between attitude toward mathematics and mathematics course work. For example, Ma (1997) found that attitude toward mathematics shows the highest association, even higher than mathematics achievement, with student mathematics course taking in the last years of high school. Therefore, different course-taking patterns can be a potential factor in the development of attitude toward mathematics.

To date, however, the literature has clearly emphasized the differential effects of various course-taking patterns in mathematics. For example, Dossey et al. (1988) used NAEP data to examine course patterns by race. On the geometry subscale, among students who had not enrolled in a geometry course, Whites scored 284 points, Hispanics scored 274 points, and Blacks scored 264 points. In contrast, among students who had enrolled in a geometry course, the results were 324, 307, and 297 points for Whites, Hispanics, and Blacks, respectively. On the algebra subscale, the scores for Whites, Hispanics, and Blacks who did not take an algebra course were 289, 276, and 273, respectively; the corresponding scores for students who took an algebra course were 328, 310, and 306. The advantage of course work in mathematics achievement was substantial regardless of racial background; it ranged from 33 to 40 points. Moreover, Jones (1987) used the HS&B data for the 1982 senior cohort to examine the relationship between mathematics course work and mathematics achievement in four gender (male and female) groups by race (White and non-White). Students with more mathematics courses as measured in Carnegie units showed higher achievement in all four groups, according to the NAEP results (Dossey et al., 1988).

Jones (1987) reported that students who had taken calculus in Grade 12 achieved better in mathematics than did those who had not taken calculus, regardless of their SES and prior mathematics ability. The National Education Longitudinal Study showed that more advanced courses at the pre-calculus level are associated with better performance in conceptual understanding and problem solving (Rock & Pollack, 1995). In light of similar findings, a legitimate ques-

tion is whether schools can use course-work policies to affect student course taking to improve mathematics performance.

A few studies have examined the mediating effect of schools on the relationship between mathematics course work and mathematics achievement. Gamoran (1987), for example, found that many school-based variables, one of which is the number of mathematics and science courses students have taken, differentiate mathematics achievement between White and Black students. He argued that schooling has different effects for Whites and Blacks. Smith (1996) used the transcript file from the HS&B to demonstrate that early (Grade 8) course work in algebra has a positive effect on long-term mathematics achievement in Grades 10 and 12. She also contended that schools can use their course offerings to influence students' academic careers and achievement. A similar argument by Bohr (1994), Hagedorn, Siadet, Nora, and Pascarella (1996) echoes at the college level.

There are, however, two limitations in the research reported to date. First, almost all studies focused on whether completing more mathematics courses contributes to better achievement in mathematics. Little is known, however, about what courses are most strongly associated with mathematics achievement. For example, if pre-algebra and Algebra I are both offered in a certain grade, do they have the same effects on mathematics achievement? Furthermore, how important is the potential effect on mathematics achievement if students take a certain course? Second, most studies discussed in this article used longitudinal data that included usually two, or at most three, waves (or collections). The lack of longitudinal data that cover the entire secondary schooling obviously limits their findings. Consequently, researchers have no empirical answers to many basic questions. For example, are there certain mathematics courses that affect mathematics achievement in multiple grades? If so, is their potential effect the same across grades? Suitable data for addressing similar issues became available only recently. The Longitudinal Study of American Youth (LSAY) presents an opportunity for a detailed examination of many issues that have been reported on the effects of mathematics course work.

Apart from high achievement, positive attitude is the other element that has been universally acclaimed as a favorable outcome of schooling (Chesler & Caves, 1981). There are few studies, however, that have described how different mathematics course work contributes to the development of attitude toward mathematics. Many basic questions remain unanswered. For instance, does mathematics course work affect attitude toward mathematics? Do certain mathematics courses affect attitude toward mathematics more substantially than others? Are there mathematics courses that affect attitude toward mathematics at multiple-grade levels? Those issues are among the concerns that I addressed in the current study.

Two specific questions were examined in this research. The first question was whether there are mathematics

courses that promote mathematics achievement and attitude toward mathematics more significantly than others. The significance of that research question is that certain mathematics courses, particularly advanced courses, contain more critical elements than others to develop mathematical skills and experiences. Completing those courses results in greater cognitive and affective preparation and improvement. The second research question was whether certain mathematics courses can affect mathematics achievement and attitude toward mathematics in multiple grades in which the courses are offered. The significance of that research question is that if a certain mathematics course is critical, then greater cognitive and affective improvement are expected no matter when students take the course.

With a better national sample (covering the entire secondary grades), I reexamined the conclusion that the effect of mathematics course work on mathematics achievement is over and above the effect of student background characteristics. The above research questions were tested with statistical controls over student background characteristics such as gender, SES, and student prior mathematics performance. In parallel, I also examined the effects of mathematics course work on attitude toward mathematics.

Method

Data

The data in this study were taken from the LSAY, a national 6-year panel study of mathematics and science education in public middle and high schools in the United States (Miller & Hoffer, 1994). The LSAY examines two sets of public schools—a national probability sample of 52 schools and a special sample of 8 schools in districts with outstanding elementary science programs. The LSAY originated in the fall of 1987 with samples of about 60 seventh graders (Cohort 2) and 60 tenth graders (Cohort 1) from each of 60 localities across the United States. The 7th and 10th graders were followed for 6 years. The analysis employed the Cohort 2 data (7th grade to 12th grade). The LSAY sample contained 3,116 students. Initial sample sizes in the analysis were 3,116 students in 7th grade; 2,798 in 8th grade; 2,748 in 9th grade; 2,583 in 10th grade; 2,409 in 11th grade; and 2,215 in 12th grade. The decrease in resultant sample size was caused by missing data on the student questionnaire and absence from school for various reasons (e.g., sickness, emigration, dropout).

Variables

Measures of mathematics course work. These measures (5 in the seventh grade, 8 in the eighth grade, 10 in the ninth grade, 12 in the tenth grade, and 13 in the eleventh grade) were derived from the LSAY composite variable measuring the highest mathematics course that each student took in each grade. The variable included all the possible courses

that students could take as their most advanced courses in a certain grade. Following Hoffer (1997), a number of dummy variables were created from the variable with the category “no course” as the reference. For example, pre-algebra was a dichotomous variable in Grade 7. The effect of that variable was compared with that of the reference variable (no course).

Measure of achievement. Mathematics achievement tapped three skill dimensions with 60 items: simple recall and recognition, routine problem solving, and more complicated problem solving. Cronbach's alpha was .86, .91, .92, .94, .95, and .95 from Grade 7 through Grade 12, respectively. The test scores were formula scores that were adjusted for difficulty, reliability, and guessing on the basis of item response theory. As a result, the test scores could be compared across test forms and grade levels.

Measure of attitude. Attitude toward mathematics was a composite variable according to a scale of nine items intended to measure four components of attitude: interest, utility, ability, and anxiety. The scale was constructed such that higher values indicated more positive attitude. Cronbach's alpha was .69, .66, .67, .72, .76, and .74 from Grade 7 through Grade 12, respectively.

Control variables. Initially, student background characteristics included gender, SES, age, and number of parents and siblings. A measure of race-ethnicity was not available in the LSAY; however, the presence of SES may, in part, compensate this loss because the literature shows that the effect of race-ethnicity usually subsides once SES is included in the analysis (Rumberger, 1983). Unfortunately, the number of parents and siblings could not be used in this analysis because of substantial missing data, particularly in the case of number of parents (the variable measuring marital status, from which the number of parents was derived, had over 30% missing data). Among remaining variables, gender and SES were used as time-invariant variables, whereas age was used as a time-varying variable. *Gender*, renamed *female*, was a dummy variable coded 1 for females and 0 for males. SES was in a standardized scale with a mean of 0 and a standard deviation of 1. The unit of age was set as *month* in this analysis.

Statistical Procedures

I used multiple regression/correlation (MRC) techniques to estimate the effects of mathematics course work on mathematics achievement and attitude toward mathematics. Cohen and Cohen (1983) described three different methods through which independent variables can be entered into a regression equation: simultaneous; stepwise; and block (hierarchical) methods. Regression, using the block method, is considered theoretically conservative, statistically rigorous, and practically suitable for explanatory studies because it ensures that the influence of preceding sets of independent variables is statistically controlled to obtain good validity for a follow-up set of independent variables

(Cohen & Cohen, 1983). I used that type of regression techniques in the current analysis.

Measures of achievement and attitude in a certain grade were the dependent variables and those in the previous grade were used as prior measures. I entered prior achievement (or attitude) first into the equation, followed by the set of control variables. Finally, I entered the set of course indicators. The significance of a set of independent variables was determined by the increment R^2 for the set over and above the R^2 for the set(s) entered earlier. That procedure was to adjust for student background characteristics. However, complete control over student background variables (e.g. prior attainment, social-demographic variables) is not possible in a nonexperimental, nonrandomized design. Statistical control was a partial adjustment for selected student background characteristics in this analysis.

Specifically, two regression equations were assessed at each level from Grades 8 to 12. For example, in 1988 when students were in Grade 8, one equation regressed the 1988 mathematics achievement scores on the 1987 mathematics achievement scores, the control variables, and the course-work indicators (low Grade 7 mathematics, average Grade 7 mathematics, high Grade 7 mathematics, pre-algebra, and Algebra I). Because the 1987 achievement scores were used as one of the independent variables, the effects of other independent variables signified the impact of each variable on how much student test scores had changed from 1987 to 1988. That is appropriate given that the major purpose of this analysis was to examine the effect of mathematics course work on changes in mathematics achievement. Note that course work refers to those courses that students chose to take in 1987 (Grade 7) rather than in 1988 (Grade 8) because it is appropriate to assume that student course work in Grade 7 affected their achievement in Grade 8. Thus, results of those regression analyses would estimate how many changes (in test scores), if any, could be attributed to completing a certain course. In parallel, the other equation regressed the 1988 mathematics attitude scores on the 1987 mathematics attitude scores, the control variables, and the course-work indicators, concentrating on the effect of mathematics course work on attitude changes.

I used effect size to indicate the relative importance of each course indicator. The standardized regression coefficient can be considered a measure of effect size because it indicates the change (in terms of percentage of a standard deviation) in the dependent variable that is associated with one unit change in an independent variable, with other variables in the equation statistically controlled (Cohen & Cohen, 1983). In that analysis, the coefficient estimated the change in terms of a percentage of a standard deviation in, for example, mathematics achievement if a nonparticipant in a certain course had taken that course, with other variables in the equation held constant. The effects of different course indicators on achievement could then be compared.

The course-work effects also can be appreciated from a different, perhaps more meaningful, perspective. Because

the course-work indicators were all dichotomous variables, I used the percentage of distribution nonoverlap, or U_3 statistic (Cohen, 1988, p. 29), to denote the percentage of the group of students who did not take a certain course that was exceeded by half of the students in the group who did take the course.² Because this analysis standardized the course work effects, the U_3 statistic also indicated the expected change in scores or percentiles associated with taking a certain course.

Statistical Problems and Solutions

In terms of research design, the current analysis was equivalent to a multiple-factor analysis of covariance (ANCOVA) in which the course indicators were factors and prior achievement (or attitude) and the control variables were covariates. The problem with course indicators as factors was that there were legitimate empty cells in the factorial design. Mathematical knowledge is highly sequential; thus, it is unlikely, for example, that there are students who studied trigonometry but did not study geometry. Coupled with the unequal numbers of students taking different courses (disproportional cell frequencies), the empty cells created statistical problems in estimation. Kirk (1982) suggested that the general linear model (GLM) approach should be used to cope with disproportional cell frequencies with empty cells. I used Kirk's suggestion to implement both GLM and MRC so that the interpretation of MRC results would be backed up by the GLM results. Furthermore, Lee and Bryk (1988) suggested the use of multiple statistical techniques in the presence of statistical uncertainty. In this analysis, I used multiple statistical methods, including descriptive comparison, correlation analysis, GLM, and MRC, to triangulate any statistical inferences regarding the effects of course work on attainment in mathematics.

Results

I performed 10 regression analyses to examine the effects of mathematics course work on mathematics achievement and attitude toward mathematics. Tables 1–10 represent the final statistical models. Interaction effects between significant course indicators and prior measures and control variables were tested in each regression. Non-significant interactions were deleted to simplify the models. I compared the results of the final statistical models with those from correlation analyses (reported in the tables) and ANCOVA (not reported). In all cases, I identified the same set of course-work indicators as having the strongest effects on mathematics achievement and attitude toward mathematics.

Tables 1 and 2 display descriptive and inferential information on the effects of different mathematics course work in 1987 (Grade 7) on achievement in mathematics and attitude toward mathematics in 1988 (Grade 8). Students in pre-algebra and Algebra I showed the largest increases in

Table 1.—Means and Standard Deviations on 1987 and 1988 Mathematics Achievement and Attitude Toward Mathematics, by Courses Taken in 1987

Course	1987 mathematics achievement		1988 mathematics achievement		1987 attitude toward mathematics		1988 attitude toward mathematics	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
	No course	37.73	4.89			9.48	1.20	
Low Grade 7 mathematics	40.91	7.07	41.71	8.73	10.81	2.77	10.67	2.53
Average Grade 7 mathematics	48.84	8.59	50.45	9.77	11.01	2.71	11.24	2.72
High Grade 7 mathematics	55.25	8.96	56.44	8.92	11.01	3.00	11.07	2.74
Pre-algebra	60.11	8.07	62.40	10.02	11.80	2.48	11.59	2.47
Algebra I	67.27	5.53	69.20	8.71	13.51	2.15	13.66	1.72

Note. Calculations were based on $n = 3,116$ students in 1987 who had course-taking information. Mathematics (NEC) and honors geometry contained a single student (case) and were eliminated in all statistical analyses.

Table 2.—Estimated Effects of Mathematics Course Work in 1987 on Achievement in Mathematics and Attitude Toward Mathematics in 1988

Independent variable ^a	<i>R</i> ² change ^b	Achievement in mathematics ^c			Attitude toward mathematics ^d		
		ES	<i>r</i>	<i>U</i> ₃	ES	<i>r</i>	<i>U</i> ₃
1987 mathematics achievement score	.221***	.582***	.702				
1987 mathematics attitude score	.268***				.525***	.530	
Female (vs. male)	.003***	.035*	.090	51.60	-.034*	-.046	48.80
Socioeconomic status	.002*	.040**	.280		.027	.083	
Age of student		-.021	-.250		.020	-.039	
Low Grade 7 mathematics (vs. no course)	.023***	.189	-.334	—	-.059***	-.070	47.61
Average Grade 7 mathematics (vs. no course)	.004*	.406	-.146	—			
High Grade 7 mathematics (vs. no course)		.289	.133	—	-.005	-.021	—
Pre-algebra (vs. no course)		.427*	.389	66.64	-.007	.057	—
Algebra I (vs. no course)		.106*	.118	54.38	.032	.068	—
Variance explained (adjusted)			52%			28.5%	

Note. ES = effect size; *r* = correlation coefficient. *U*₃ denotes the proportion of the nonparticipant group that is exceeded by 50% of the individuals in the participant group (Cohen, 1988).

^aIndependent variables are blocked to show the different sets of variables entered into the regression. ^bBold numbers indicate the *R*² increment associated with the sets of variables for the regression on achievement, whereas regular numbers indicate the *R*² increment for the regression on attitude.

^cIn the model of achievement in mathematics, the variables denoting mathematics (NEC) and honors geometry were excluded because they contained a single student (case). ^dIn the model of attitude toward mathematics, the variable denoting average Grade 7 mathematics was excluded due to collinearity. The variables denoting mathematics and honors geometry were excluded from the model because they contained a single student (case).

* $p < .05$. ** $p < .01$. *** $p < .001$.

means of mathematics achievement from Grades 7 to 8. Relative to the two statistically significant course work indicators, pre-algebra had a stronger effect on mathematics achievement (effect size = .43, $p < .05$). Students with pre-algebra in Grade 7 scored almost half a standard deviation higher in mathematics achievement in Grade 8 than those without any mathematics course in Grade 7. The *U*₃ statistic indicates that 67% of the no-course students were exceeded by half of the students with pre-algebra as their most advanced course, suggesting that completing pre-algebra was associated with an improvement in the average students' mathematics achievement from the 50th percentile to the 67th percentile. That observed effect size (.43) was both statistically and substantively substantial. Consider

SAT scores (σ is about 100): In a population of students with a mean SAT score of 500, those with pre-algebra in Grade 7 would score 543 in Grade 8. That effect size becomes more substantial given that the model included controls over student background (gender, age, and, most important, SES and prior achievement).

Algebra I also had a statistically significant effect on mathematics achievement (effect size = .11, $p < .05$). Students who took Algebra I in Grade 7 achieved more than one tenth of a standard deviation higher in mathematics in Grade 8 than those who did not take mathematics in Grade 7. Completing Algebra I was associated with an improvement in average students' mathematics achievement from the 50th to the 54th percentile. Although the effect size of

Algebra I was smaller than that of pre-algebra, it can still be considered practically substantial especially because the model included controls over individual characteristics and academic background. Consider SAT scores with a mean of 500: Students who studied Algebra I in Grade 7 would score 511 in Grade 8. Finally, students who took courses other than pre-algebra and Algebra I in Grade 7 did not perform any better in mathematics in Grade 8 than those who did not study mathematics in Grade 7.

Except for low seventh-grade mathematics, no course-work indicators in Grade 7 had statistically significant effects on attitude toward mathematics in Grade 8. Students who took low seventh-grade mathematics showed statistically more negative attitude in Grade 8 than those who did not enroll in mathematics in Grade 7. That observation may not be practically important, however, because it represents a drop of .06 of a standard deviation in average participants' attitude or a drop from the 50th to the 48th percentile in terms of U_3 statistic. The course-work model for achievement in mathematics explained more than half of the variance in mathematics achievement (52%). The model for attitude toward mathematics accounted for less than one third of the variance in mathematics attitude (29%). Therefore, the course-work model predicted mathematics achievement much better than attitude toward mathematics. If course work is associated with achievement in mathematics, it is less the case for attitude toward mathematics.

From 1988 to 1989 (Grade 8 to Grade 9), students in geometry, pre-algebra, Algebra I, and Algebra I Honors demonstrated the largest increases in means of achievement in mathematics (Table 3). Table 4 shows that, among statistically significant course indicators, Algebra I had the strongest effect on mathematics achievement, with an effect size of .28 ($p < .05$), followed by pre-algebra, Algebra I Honors, and geometry (effect size = .22, .18, and .04,

respectively). Although the effect of geometry on mathematics achievement may not be practically important, the other three course indicators were more substantial. Students who took Algebra I, pre-algebra, and Algebra I Honors in Grade 8 scored .28, .22, and .18, respectively, of a standard deviation higher in mathematics achievement in Grade 9 than those who did not study mathematics in Grade 8. In terms of SAT scores with a mean of 500, students who took Algebra I, pre-algebra, and Algebra I Honors in Grade 8 would score 528, 522, and 518, respectively, in Grade 9. Completing Algebra I, pre-algebra, and Algebra I Honors was associated with an improvement in average students' mathematics achievement from the 50th to the 61st percentile, from the 50th to the 59th percentile, and from the 50th to the 57th percentile, respectively.

There was a statistically significant interaction in the model ($p < .05$; see Table 4). First, for students who did not take Algebra I in Grade 8, male and female students performed equally in mathematics achievement in Grade 9. Second, among students who took Algebra I as their most advanced course in Grade 8, boys performed statistically better in mathematics achievement in Grade 9 than did girls. However, with a measure about .04 of a standard deviation, the gender gap may not be practically substantial. All of the mathematics course-work indicators were statistically non-significant for attitude toward mathematics except for geometry, which showed a practically small, although statistically significant, effect size about .05 of a standard deviation. Therefore, mathematics course work did not have significant effects on mathematics attitude. The course-work model predicted mathematics achievement far better (56% of the variance) than did attitude toward mathematics (29% of the variance).

Tables 5 and 6 show the effects of different mathematics course work in Grade 9 on achievement in mathematics and attitude toward mathematics in Grade 10. From Grade

Table 3.—Means and Standard Deviations on 1988 and 1989 Mathematics Achievement and Attitude Toward Mathematics, by Courses Taken in 1988

Course	1988 mathematics achievement		1989 mathematics achievement		1988 attitude toward mathematics		1989 attitude toward mathematics	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
No course	44.88	11.82	41.77	13.24	10.67	1.88	9.42	3.10
Low Grade 8 mathematics	41.30	8.67	44.00	9.47	10.52	2.58	10.17	2.44
Average Grade 8 mathematics	47.80	8.80	51.42	10.19	11.01	2.63	10.50	2.47
Mathematics (NEC)	51.20	10.57	53.81	11.23	10.83	2.62	10.31	2.96
Geometry	68.04	0.15	71.30	0.68	14.00	0.00	14.36	1.34
Pre-algebra	52.17	9.58	56.24	10.97	11.24	2.71	11.24	2.57
Algebra I	63.00	9.38	67.93	9.39	11.99	2.59	11.99	2.25
Algebra I Honors	63.88	9.48	69.52	9.70	12.31	2.48	12.31	2.89
Algebra II Honors	75.98	3.56	75.17	4.64	12.20	2.36	12.20	2.43

Note. Calculations were based on $n = 2,794$ students in 1988 who had course-taking information. Consumer mathematics and Algebra II contained a single student (case) and were eliminated in all statistical analyses.



Table 4.—Estimated Effects of Mathematics Course Work in 1988 on Achievement in Mathematics and Attitude Toward Mathematics in 1989

Independent variable ^a	<i>R</i> ² change ^b	Achievement in mathematics ^c			Attitude toward mathematics ^c		
		ES	<i>r</i>	<i>U</i> ₃	ES	<i>r</i>	<i>U</i> ₃
1988 mathematics achievement score	.226***	.579***	.720				
1988 mathematics attitude score	.256***				.512***	.526	
Female (vs. male)	.009***	.010	.070	—	-.037	-.049	—
Socioeconomic status	.004*	.064**	.295		.050*	.104	
Age of student		-.070***	-.273		-.016	-.056	
Low Grade 8 mathematics (vs. no course)	.028***	.034	-.356	—	.010	-.088	—
Average Grade 8 mathematics (vs. no course)	.008**	.109	-.156	—	.023	-.061	—
Mathematics (NEC) (vs. no course)		.063	-.038	—	-.011	-.056	—
Geometry (vs. no course)		.041*	.068	51.60	.052*	.077	51.99
Pre-algebra (vs. no course)		.218*	.033	58.71	.090	.034	—
Algebra I (vs. no course)		.276***	.351	61.03	.089	.104	—
Algebra I Honors (vs. no course)		.180***	.242	57.14	.025	.039	—
Female × Algebra I		-.042					
Variance explained (adjusted)			56.3%			28.9%	

Note. ES = effect size; *r* = correlation coefficient. *U*₃ denotes the proportion of the nonparticipant group that is exceeded by 50% of the individuals in the participant group (Cohen, 1988).

^aIndependent variables are blocked to show the different sets of variables entered into the regression. ^bBold numbers indicate the *R*² increment associated with the sets of variables for the regression on achievement, whereas regular numbers indicate the *R*² increment for the regression on attitude.

^cThe variables denoting consumer mathematics and Algebra II were excluded from the model because they contained a single student (case).

p* < .05. *p* < .01. ****p* < .001.

9 to Grade 10, no course-work indicator had any significant effect on either mathematics achievement or mathematics attitude. That does not mean, however, that the course-work model did not fit the data. In comparison with previous models, this model explained more variance in both mathematics achievement and mathematics attitude. Nearly two thirds of the variance was accounted for in mathematics achievement (62%) and one third of the variance was accounted for in mathematics attitude (33%). That situation may be attributable to factors related to the transition from junior to senior high school, which requires many cognitive and affective adjustments. Variables related to those adjustments may diminish the potential effects of course work on mathematics achievement and attitude toward mathematics.

Tables 7 and 8 present changes (from Grade 10 to Grade 11) in mathematics achievement and attitude toward mathematics under different mathematics course work. Four course-work indicators were statistically significant (*p* < .05), three of which showed negative effects. Students who studied basic mathematics, consumer mathematics, and pre-algebra in Grade 10 scored .06 of a standard deviation less in mathematics achievement in Grade 11 than those who did not study mathematics in Grade 10. In terms of SAT scores with a mean of 500, students with those courses in Grade 10 would score 494 in Grade 11. Nevertheless, the negative effects do not appear to be practically significant. Algebra II had a statistically significant, positive effect. Students with that course in Grade 10 achieved .09 of a standard deviation higher in mathematics in Grade 11 than did

those without course work in Grade 10. That result corresponds to an SAT score of 509. As an equivalent, completing Algebra II was associated with improved average students' mathematics achievement from the 50th to the 53rd percentile. Such an effect may be marginally important in educational practice. The trend in Grades 9 and 10 continued in terms of attitude toward mathematics in Grades 10 and 11. Mathematics course work in Grade 10 did not appear to have significant effects on mathematics attitude in Grade 11. The course-work model explained about 60% of the variance in mathematics achievement and about 39% of the variance in mathematics attitude.

Finally, Tables 9 and 10 list estimated effects of mathematics course work in 1991 (Grade 11) on achievement in mathematics and attitude toward mathematics in 1992 (Grade 12). Table 10 shows a harvest of statistically significant course-work indicators that affected mathematics achievement (*p* < .05). Trigonometry, Algebra II, analytic geometry, geometry, and calculus all had effect sizes above .10 of a standard deviation. For example, students who took trigonometry as their most advanced course in Grade 11 scored nearly one fifth of a standard deviation higher in mathematics achievement in Grade 12 than those who did not take mathematics courses in Grade 11. Consider SAT scores with a mean of 500: Students with trigonometry in Grade 11 would score 518 in Grade 12. Completing trigonometry was associated with an improvement in average students' mathematics achievement from the 50th to the 57th percentile. That improvement was substantial because the course-work model contained powerful control vari-

Table 5.—Means and Standard Deviations on 1989 and 1990 Mathematics Achievement and Attitude Toward Mathematics, by Courses Taken in 1989

Course	1989 mathematics achievement		1990 mathematics achievement		1989 attitude toward mathematics		1990 attitude toward mathematics	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
	No course	58.08	14.68	50.90	18.80	8.64	1.96	8.74
Basic mathematics	44.46	10.61	46.79	10.61	10.50	2.54	10.29	2.78
Consumer mathematics	46.44	9.05	42.22	10.54	9.63	2.24	9.28	2.53
Mathematics (NEC)	47.98	9.85	49.09	10.53	10.37	2.54	10.33	2.12
Geometry	68.82	9.82	72.69	12.15	11.31	2.44	11.75	2.51
Honors geometry	70.72	8.12	75.29	7.71	11.74	2.36	11.36	2.66
Pre-algebra	51.02	9.40	54.22	10.80	10.47	2.48	9.68	3.04
Algebra I	57.76	10.55	61.42	11.82	10.95	2.65	10.39	2.80
Algebra I Honors	62.73	11.52	64.04	12.25	10.56	3.02	10.30	2.69
Algebra II	68.39	11.71	71.11	11.87	12.05	2.87	11.62	2.66
Algebra II Honors	71.80	7.42	75.45	7.43	11.33	2.41	11.87	1.91

Note. Calculations were based on $n = 2,740$ students in 1989 who had course-taking information. Vocational mathematics contained a single student (case) and was eliminated in all statistical analyses.

Table 6.—Estimated Effects of Mathematics Course Work in 1989 on Achievement in Mathematics and Attitude Toward Mathematics in 1990

Independent variable ^a	<i>R</i> ² change ^b	Achievement in mathematics ^c			Attitude toward mathematics ^c		
		ES	<i>r</i>	<i>U</i> ₃	ES	<i>r</i>	<i>U</i> ₃
1989 mathematics achievement score	.257***	.646***	.770	—	—	—	—
1989 mathematics attitude score	.278***	—	—	.535***	.550	—	—
Female (vs. male)	.003***	-.024	.043	—	-.056**	-.091	52.39
Socioeconomic status	.004**	.020	.276	—	.041*	.045	—
Age of student		-.054***	-.267	—	-.017	-.052	—
Basic mathematics (vs. no course)	.020***	-.056	-.375	—	-.095	-.023	—
Consumer mathematics (vs. no course)	.024***	-.053	-.150	—	-.053	-.049	—
Mathematics (NEC) (vs. no course)		-.018	-.130	—	-.032	-.004	—
Geometry (vs. no course)		.115	.287	—	.031	.136	—
Honors geometry (vs. no course)		.091	.220	—	-.019	.071	—
Pre-algebra (vs. no course)		.004	-.162	—	-.168	-.106	—
Algebra I (vs. no course)		.075	.131	—	-.155	-.027	—
Algebra I Honors (vs. no course)		.023	.061	—	-.050	-.013	—
Algebra II (vs. no course)		.037	.102	—	-.005	.055	—
Algebra II Honors (vs. no course)		.045	.137	—	.024	.064	—
Variance explained (adjusted)			61.9%			32.5%	

Note. ES = effect size; *r* = correlation coefficient. *U*₃ denotes the proportion of the nonparticipant group that is exceeded by 50% of the individuals in the participant group (Cohen, 1988).

^aIndependent variables are blocked to show the different sets of variables entered into the regression. ^bBold numbers indicate the *R*² increment associated with the sets of variables for the regression on achievement, whereas regular numbers indicate the *R*² increment for the regression on attitude.

^cThe variable denoting vocational mathematics was excluded from the model because it contained a single student (case).

* $p < .05$. ** $p < .01$. *** $p < .001$.

ables such as SES and prior mathematics achievement. Note that the combine effects of course work can be substantial. For example, the combined effect size for Algebra II and trigonometry was .34, and for trigonometry and calculus, it was .29.

The interaction term between 1991 (Grade 11) mathematics achievement and analytic geometry was statistically significant ($p < .05$). Among students who took analytic

geometry, those with higher achievement in Grade 11 achieved significantly better mathematics scores in Grade 12 than those with lower achievement in Grade 11. In terms of SAT scores, if two participants in analytic geometry were 100 (one standard deviation) scores apart in Grade 11, they would be 168 scores apart in Grade 12 (effect size = $.55 + .13 = .68$). That finding implies that students with high prior achievement may benefit more

Table 7.—Means and Standard Deviations on 1990 and 1991 Mathematics Achievement and Attitude Toward Mathematics, by Courses Taken in 1990

Course	1990 mathematics achievement		1991 mathematics achievement		1990 attitude toward mathematics		1991 attitude toward mathematics	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
	No course	52.13	16.14	54.97	14.67	9.62	3.06	9.75
Basic mathematics	44.93	9.97	47.12	11.15	10.59	2.56	9.78	2.34
Vocational mathematics	48.84	11.25	53.51	10.78	10.55	2.20	9.83	2.27
Consumer mathematics	47.81	11.31	48.39	11.98	10.22	2.88	9.71	2.38
Geometry	62.51	11.19	64.04	12.92	10.16	2.76	10.29	2.95
Honors geometry	71.58	8.07	73.31	11.22	10.97	2.80	10.36	2.93
Pre-algebra	49.40	9.67	49.82	11.37	9.80	2.79	9.45	2.51
Algebra I	54.70	11.43	56.43	12.50	10.22	2.91	9.39	2.82
Algebra I Honors	57.48	12.24	55.71	14.31	9.96	2.98	8.36	2.94
Algebra II	68.75	12.43	72.01	12.32	11.33	2.75	11.13	2.72
Algebra II Honors	74.84	9.36	77.80	9.82	11.86	2.24	11.70	2.58
Trigonometry	73.35	4.30	75.16	8.71	13.17	2.42	12.27	1.95
Analytic geometry	88.39	4.45	81.76	8.82	14.07	1.94	12.73	1.92

Note. Calculations were based on $n = 2,573$ students in 1990 who had course-taking information. Trigonometry honors contained a single student (case) and was eliminated in all statistical analyses.

Table 8.—Estimated Effects of Mathematics Course Work in 1990 on Achievement in Mathematics and Attitude Toward Mathematics in 1991

Independent variable ^a	R^2 change ^b	Achievement in mathematics ^c			Attitude toward mathematics ^c		
		ES	r	U_3	ES	r	U_3
1990 mathematics achievement score	.262***	.652***	.761	—	.587***	.609	—
1990 mathematics attitude score	.323***	—	—	—	—	—	52.39
Female (vs. male)	.000	-.001	.034	—	-.011	-.077	—
Socioeconomic status	.001	.001	.247	—	.032	.096	—
Age of student	—	.001	-.223	—	-.017	-.077	—
Basic mathematics (vs. no course)	.022***	-.062*	-.239	47.61	-.004	-.028	—
Vocational mathematics (vs. no course)	.019***	-.003	-.070	—	-.019	-.031	—
Consumer mathematics (vs. no course)	—	-.064*	-.204	47.61	.001	-.034	—
Geometry (vs. no course)	—	.033	.111	—	.075	.045	—
Honors geometry (vs. no course)	—	.057	.184	—	.010	.018	—
Pre-algebra (vs. no course)	—	-.062*	-.181	47.61	-.002	-.044	—
Algebra I (vs. no course)	—	-.046	-.208	—	-.059	-.156	—
Algebra I Honors (vs. no course)	—	-.032	-.042	—	-.037	-.058	—
Algebra II (vs. no course)	—	.075	.248	—	.053	.103	—
Algebra II Honors (vs. no course)	—	.081*	.256	53.19	.068	.144	—
Trigonometry (vs. no course)	—	.020	.063	—	.021	.059	—
Analytic geometry (vs. no course)	—	.011	.078	—	.018	.054	—
Variance explained (adjusted)	—	—	59.8%	—	—	39.0%	—

Note. ES = effect size; r = correlation coefficient. U_3 denotes the proportion of the nonparticipant group that is exceeded by 50% of the individuals in the participant group (Cohen, 1988).

^aIndependent variables are blocked to show the different sets of variables entered into the regression. ^bBold numbers indicate the R^2 increment associated with the sets of variables for the regression on achievement, whereas regular numbers indicate the R^2 increment for the regression on attitude.

^cThe variable denoting trigonometry honors was excluded from the model because it contained a single student (case).

* $p < .05$. ** $p < .01$. *** $p < .001$.

from taking analytic geometry than do students with low prior achievement.

Mathematics course work in Grade 11 showed no practically significant effects on attitude toward mathematics in Grade 12, although there were statistically significant

effects associated with honors geometry, trigonometry, analytic geometry, and calculus (effect sizes from .05 to .07). Students who took honors geometry in Grade 11, for example, scored about .06 of a standard deviation higher in mathematics attitude than students who did not take

Table 9.—Means and Standard Deviations on 1991 and 1992 Mathematics Achievement and Attitude Toward Mathematics, by Courses Taken in 1991

Course	1991 mathematics achievement		1992 mathematics achievement		1991 attitude toward mathematics		1992 attitude toward mathematics	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
	No course	54.47	12.96	52.38	13.13	9.18	2.63	9.45
Basic mathematics	45.12	9.31	45.40	10.12	9.71	2.62	8.93	2.38
Vocational mathematics	53.03	12.54	52.82	13.02	10.62	2.34	9.60	2.72
Consumer mathematics	49.51	12.10	47.10	12.08	10.27	2.69	10.30	2.51
Geometry	59.19	13.58	59.61	12.87	9.66	2.90	9.68	2.62
Honors geometry	61.43	14.31	58.26	19.85	10.42	2.52	11.59	2.31
Pre-algebra	44.29	10.69	50.33	10.59	9.62	2.41	10.58	2.90
Algebra I	52.16	11.51	52.67	11.50	9.58	2.76	9.37	2.60
Algebra II	65.37	11.45	64.25	13.21	10.29	2.94	10.11	2.77
Algebra II Honors	74.26	9.26	74.26	9.44	10.03	2.84	9.42	2.80
Trigonometry	70.72	11.67	72.33	12.00	10.59	2.94	10.58	2.79
Trigonometry Honors	74.91	7.20	75.92	9.04	11.07	2.94	10.59	2.56
Analytic geometry	77.15	10.83	77.22	12.72	11.65	2.44	11.57	2.35
Calculus	80.11	6.43	84.31	5.73	12.66	1.92	12.47	2.48

Note. Calculations were based on $n = 2,635$ students in 1991 who had course-taking information. Algebra Honors contained a single student (case) and was eliminated in all statistical analyses.

Table 10.—Estimated Effects of Mathematics Course Work in 1991 on Achievement in Mathematics and Attitude Toward Mathematics in 1992

Independent variable ^a	<i>R</i> ² change ^b	Achievement in mathematics ^c			Attitude toward mathematics ^c		
		ES	<i>r</i>	<i>U</i> ₃	ES	<i>r</i>	<i>U</i> ₃
1991 mathematics achievement score	.170***	.547***	.723				
1991 mathematics attitude score	.325***				.593***	.622	
Female (vs. male)	.004**	.009	.018	—	-.056*	-.076	47.61
Socioeconomic status	.004*	.004	.228		-.035	.059	
Age of student		.066***	-.246		-.032	-.089	
Basic mathematics (vs. no course)	.042***	-.009	-.176	—	-.012	-.053	—
Vocational mathematics (vs. no course)	.019***	.027	-.066	—	-.018	-.017	—
Consumer mathematics (vs. no course)		-.026	-.227	—	.022	.014	—
Geometry (vs. no course)		.109***	-.073	54.38	.000	-.089	—
Honors geometry (vs. no course)		.008	-.029	—	.057**	.050	52.39
Pre-algebra (vs. no course)		.045*	-.091	51.99	.010	-.020	—
Algebra I (vs. no course)		.021	-.189	—	-.027	-.096	—
Algebra II (vs. no course)		.166***	.097	56.75	.053	.001	—
Algebra II Honors (vs. no course)		.088***	.110	53.59	.003	-.043	—
Trigonometry (vs. no course)		.718***	.188	57.14	.051*	.041	51.99
Trigonometry honors (vs. no course)		.076***	.093	53.19	.012	.019	—
Analytic geometry (vs. no course)		.012**	.351	54.38	.071*	.208	52.79
Calculus (vs. no course)		.109***	.151	54.38	.045*	.089	51.99
1991 Mathematics Score × Analytic Geometry		.131***					
Female × Analytic Geometry					.060*		
Variance explained (adjusted)				57.1%			40.2%

Note. ES = effect size; *r* = correlation coefficient. *U*₃ denotes the proportion of the nonparticipant group that is exceeded by 50% of the individuals in the participant group (Cohen, 1988).

^aIndependent variables are blocked to show the different sets of variables entered into the regression. ^bBold numbers indicate the *R*² increment associated with the sets of variables for the regression on achievement, whereas regular numbers indicate the *R*² increment for the regression on attitude.

^cThe variable denoting Algebra I Honors was excluded from the model because it contained a single student (case).

p* < .05. *p* < .01. ****p* < .001.



courses in Grade 11 (an improvement in average students' mathematics attitude from the 50th to the 52nd percentile).

There was also a significant interaction effect between gender and analytic geometry in the model ($p < .05$). Analytic geometry had a stronger effect on female students (effect size = $.060 + .071 = .131$) than on male students (effect size = $.071$). Girls who took analytic geometry in the 11th grade scored about .13 of a standard deviation higher on attitude toward mathematics in the 12th grade than girls who did not take analytic geometry in the 11th grade. Furthermore, for students who took analytic geometry in the 11th grade, boys and girls had similar mathematics attitude in the 12th grade (effect size = $.060 - .056 = .004$). For students who did not take analytic geometry, female attitude was less positive than male attitude (effect size = $-.056$). However, that effect size was practically small.

Discussion

Using six waves of data (Grades 7 to 12) from the LSAY, I examined the effects of mathematics course work on achievement in mathematics and attitude toward mathematics, with some partial adjustment for prior measures of achievement and attitude and student background characteristics. In the early grades of high school (Grades 7 and 8), courses pertaining to algebra affected mathematics achievement. Pre-algebra, Algebra I, and Algebra I Honors were statistically significant, with effects ranging from .11 to .43 of a standard deviation. Those effects were practically substantial, considering that they materialized subsequent to accounting for the influence of powerful control variables such as prior mathematics achievement and SES.

Mathematics course work did not appear to play a role in mathematics achievement in the middle grades of high school (Grades 9 and 10). Only Algebra II Honors had a marginal effect of .09 of a standard deviation. The effects of mathematics course work were widespread in the later grades of high school, however. There were nine statistically significant courses, six of which had effects ranging from .11 to .18 of a standard deviation. Every advanced mathematics course had a statistically significant effect. There was also a statistically significant interaction that indicated that students with high prior achievement benefited more than those with low prior achievement from taking analytic geometry in Grade 11. Overall, the strongest effects of mathematics course work were in the early grades of high school, whereas course-work effects were more widespread in the later grades of high school.

Most mathematics courses, however, had no statistically significant effects on attitude across grades. The effects of the few courses that were statistically significant were not practically substantial (about .06 of a standard deviation). I found that girls who took analytic geometry developed more a positive attitude toward mathematics (about .13 of a standard deviation) than girls who did not take that course.

Implications

Many results of this study can be interpreted as a challenge to a number of widely held beliefs and assumptions about mathematics preparation, which for economics of expression will be loosely termed "myths" in the following discussion. This challenge is offered to promote more focused and rigorous investigations into students' mathematics course work.

Myth 1: One more course does not make a big difference in mathematics achievement. It is true that low-level mathematics courses do not make any difference in mathematics achievement (see Tables 2, 4, 6, 8, and 10). A close examination of the significant course-work indicators across grades shows that they all represent advanced courses (e.g., pre-algebra in Grade 7, Algebra I in Grade 8, and trigonometry in Grade 11). One advanced mathematics course may make a substantial difference in mathematics achievement. Pre-algebra in Grade 7, for example, had an effect of half a standard deviation on mathematics achievement in Grade 8. The importance of that course becomes even more substantial given that the course-work model included partial adjustment for prior mathematics achievement and student background characteristics. In other words, the effects of mathematics course work were over and above the effects of academic background and individual characteristics, or these latter variables were controlled in this study.

Myth 2: Advanced mathematics course work, in general, improves students' achievement in mathematics. The analysis indicates that not all advanced mathematics courses upgrade students' mathematics achievement. A careful inspection of all advanced courses across grades suggests an interesting pattern not observed in previous studies: The significant course-work indicators represent the relatively lower level courses in advanced mathematics. For example, pre-algebra is a lower course than Algebra I in Grade 7, but pre-algebra had a much stronger effect than did Algebra I. Trigonometry is a lower course than analytic geometry and calculus in Grade 11, but it was the most significant course-work indicator. Thus, this study indicates that the lower level courses in the family of advanced mathematics are more strongly associated with improvement in mathematics achievement than the relatively more advanced courses.

Myth 3: Only the most advanced mathematics courses stand the best chance to improve mathematics achievement. From the above discussion, the opposite may be true. In this study, lower level courses in the family of advanced mathematics had stronger effects on mathematics achievement. One possible explanation is the sequential feature of mathematical knowledge. The most advanced mathematics courses demand a solid mastery of not only basic mathematics courses but also courses that are relatively lower in level in the family of advanced mathematics. On the other hand, lower level courses in advanced mathematics are built on basic mathematics courses only. Students may have a

better chance to master those courses, and, thus, are more likely to improve their mathematics achievement.

Myth 4: Geometry courses train students' logical thinking and therefore improve students' mathematics achievement. In this analysis, geometry had a practically important effect only in Grade 11 (about .11 of a standard deviation), although it was available in almost every grade. Even that effect was secondary to that of many advanced courses in that grade (e.g., trigonometry and analytic geometry). I do not intend to downgrade the importance of geometry in this analysis but, instead, call for more analyses on this issue. The finding that geometry did not play a critical role in mathematics achievement may concern the geometry curriculum. There has been considerable effort to balance the geometry content between rigorous and practical. The element of logical thinking may be influenced by that balance. Perhaps the question to consider is to what extent, if any, the current geometry courses have emphasized logical thinking.

Myth 5: Mathematics course work has different effects on mathematics achievement between boys and girls. If different gender effects were found, there should have been many significant interactions between gender and significant course-work indicators. However, there was only one significant interaction (between gender and Algebra I in Grade 8; see Table 4) and that gender gap was not practically important. Thus, this analysis suggests that it is not likely that mathematics course work has different effects on mathematics achievement between boys and girls. A general statement is that mathematics course work has differential effects on mathematics achievement for different groups of students. I examined socioeconomic groups and age groups; SES and age had no significant interactions with significant course-work indicators across all grade levels. Students from different SES demonstrated similar improvement in mathematics achievement after they took the same courses. Mathematics course work also had similar effects on mathematics achievement of students of different ages.

Myth 6: High-achieving students benefit more from taking mathematics courses than low-achieving students. If that statement were true, there should have been many significant interactions between prior mathematics achievement and significant course-work indicators. However, only one such interaction appeared. Students with high prior achievement benefited more from taking analytic geometry in Grade 11 than did students with low prior achievement (see Table 10). Because that interaction appears to be isolated, the effect was likely caused by random chance. Mathematics courses seem to benefit all students, regardless of their academic background.

Myth 7: Mathematics course work affects students' attitude toward mathematics. Many educators worry about students' developing a negative attitude toward mathematics from the difficulties they experience in mathematics courses. Surprisingly, this analysis has shown that mathematics

course work did not appear to have any significant effects on attitude toward mathematics. The effects of the few significant courses were marginal practically. The effect of analytic geometry was meaningful, but only for girls; the effects were all positive. No significant, negative effects were detected. Thus, students did not seem to develop a negative attitude toward mathematics because they took certain mathematics courses. Overall, this analysis suggests that factors, rather than course work, may be responsible for changes in attitude toward mathematics.

Myth 8: Practical mathematics courses improve students' attitude toward mathematics. This analysis showed that no mathematics courses with more practical contents affected attitude toward mathematics. In other words, indicators representing practical mathematics courses were all nonsignificant across all grade levels. Students did not seem to favorably change their attitude toward mathematics because they saw the practical part of mathematics. However, because attitude toward mathematics in the LSAY was a composite of interest, utility, ability, and anxiety, the utility component may show some favorable improvement after taking practical mathematics courses if each component is analyzed separately. This analysis suggests, however, that mathematics course work did not affect general attitude toward mathematics as measured through the four components.

Myth 9: Some mathematics courses have more effects on girls', rather than on boys', attitude toward mathematics. That statement is not supported in the current study. There was only one significant interaction between gender and analytic geometry in Grade 11. That myth exists probably because a large body of research literature suggests that girls have a more negative attitude toward mathematics than boys do. This analysis, however, shows that the negative attitude of female students did not seem to come from taking mathematics courses. Instruction-related factors may be more useful to account for female students' negative attitude toward mathematics.

Recommendations for Future Research

This descriptive study, with its aforementioned statistical limitations, cannot solely unseat the common beliefs about mathematics preparation discussed in the preceding paragraphs. The significance of this study is that it has posed a significant challenge to those beliefs and, as such, represents a challenge to the community of mathematics educators to mount more refined research to reexamine those beliefs. Researchers may want to investigate school policies and practices as they mediate the effects of mathematics course work on achievement in mathematics and attitude toward mathematics, over and above the mediating effects of prior attainment and student characteristics. Researchers also may want to examine specific areas in achievement and attitude. Some mathematics course work may have a particular effect on certain elements of achievement (e.g., concepts and problem solving) and attitude (e.g., interest and

utility). Finally, this study, combined with those previously cited (e.g., Hoffer, 1997; Ma, 1997; Smith, 1996), advanced our knowledge to the point where the variables antecedent to mathematics achievement can be fashioned in causal-like path diagrams that would lend themselves to path analysis, or better, structural equation modeling.

NOTES

1. The term "partial adjustment" refers to the fact that an unbiased, comprehensive control over students' prior attainment and social-demographic characteristics is not possible in a nonexperimental, nonrandomized design to which most secondary analyses of large national samples belong. In other words, statistical control is only partially achieved in those analyses, including the current analysis.

2. Cohen's (1977, 1988) U_3 statistic is often used as an auxiliary illustration of effect size. It indicates in which percentile the typical person with the group median in the experimental group would fall if he or she were placed in the control group. Cohen (1977, 1988) pointed out that the use of the U_3 statistic relies on the assumptions of normality and homogeneity of variances. The U_3 statistic has been used primarily to illustrate the effects of dichotomous independent variables on the dependent measure (Fuchs & Fuchs, 1986).

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